THE AERODYNAMICS OF BIRD FLIGHT

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I. INTRODUCTION

This report is a summary of the work accomplished during the period February 1 - March 15, 1975 under NASA Contract No. NGR22-009-818 to Langley Research Center.

Our efforts during this period have been aimed at studying the flapping aspect of bird flight, more specifically to formulate a mathematical model to determine the aerodynamic loads on a wing in unsteady flapping motion. Unlike most aeronautical applications where thrust and drag are separate phenomena, in flapping flight the two are coupled and unseparable. In this report we will refer to the combination of thrust and drag as the streamwise force, represented by 'F_x'.

The general method we have adopted to handle this problem is the momentum theorem which was chosen to ensure that no secondary contributions, such as from leading edge suction, are lost through possible linearizations.

In Chapter II general forms of the momentum theorem are presented. To demonstrate the usefulness of the momentum theorem, the induced drag of an arbitrary wing in steady flow is determined by this method (Section 2.1), which is the same as the result obtained by other methods.

Due to the complexity of the 3-D unsteady problem, first the 2-D unsteady problem is undertaken. We are currently working on this problem, an outline of which is presented in Section 2.2. Also an outline of the proposed method to analyze the 3-D unsteady problem is given in Chapter III.

II. MOMENTUM THEOREM

The momentum theorem for the control volume shown in Fig. 1 is readily derived in the form

$$\sum_i \vec{F}_i = \iiint_V \frac{\partial}{\partial t} (\rho\vec{Q})\, dV + \oiint_{S+\sigma} \rho\vec{Q}\, (\vec{Q} \cdot \vec{n})\, dS. \tag{2.1}$$

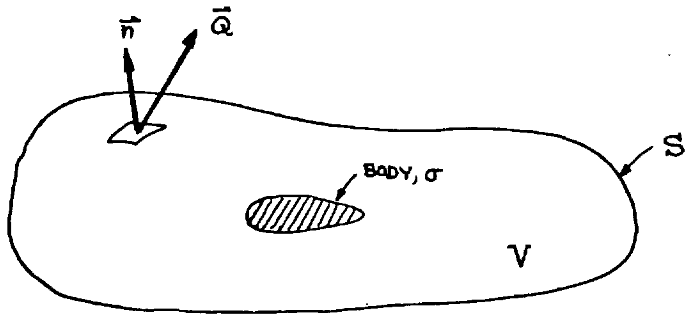

FIG. 1

For inviscid flow with small body forces, the left hand side is given by

$$\sum_i \vec{F}_i = \text{reaction forces} + \text{pressure forces}$$

$$= - \vec{F}_{body} - \oint\!\!\oint_S p \, \vec{n} \, dS. \tag{2.2}$$

Assuming an impermeable body, Eq. 2.1 becomes

$$\vec{F}_{body} = - \oint\!\!\oint_S [P\vec{n} + \rho\vec{Q}(\vec{Q} \cdot \vec{n})] \, dS - \iiint_V \frac{\partial}{\partial t} (\rho\vec{Q}) \, dV. \tag{2.3}$$

For steady flow this reduces to

$$\vec{F}_{body} = - \oint\!\!\oint_S [P \, \vec{n} + \rho\vec{Q}(\vec{Q} \cdot \vec{n})] \, dS \tag{2.4}$$

2.1 Steady Flow-Induced Drag of Arbitrary Wing

Equation 2.4 is an expression for the force experienced by a body immersed in an inviscid steady flow. This result can be specialized for the case of a uniform parallel free stream U_∞ . Introducing the non-dimensional perturbation velocity

$$\vec{Q} = U_\infty \; (\vec{q} + \vec{i}) \tag{2.5}$$

Equation 2.4 becomes

$$\vec{F}_{body} = -\oiint_S [P\,\vec{n} + \rho U_\infty{}^2(\vec{q} + \vec{i})(\vec{q}\cdot\vec{n} + \vec{i}\cdot\vec{n})]\, dS. \tag{2.6}$$

This result can be further simplified with the aid of continuity equation.

$$\vec{F}_{body} = -\oiint_S [(P - P_\infty)\vec{n} + \rho U_\infty{}^2\,\vec{q}(\vec{q}\cdot\vec{n} + \vec{i}\cdot\vec{n})]\, dS \tag{2.7}$$

where insertion of P_∞ into this equation does not change the result since

$$\oiint_S P_\infty\,\vec{n}\, dS = 0\,.$$

Now, we will proceed to determine the induced drag (streamwise component of the force on the body) on an arbitrary wing in a uniform, parallel, constant density ($\rho = \rho_\infty$) flow. The control volume and the coordinate axes are shown in Fig. 2.

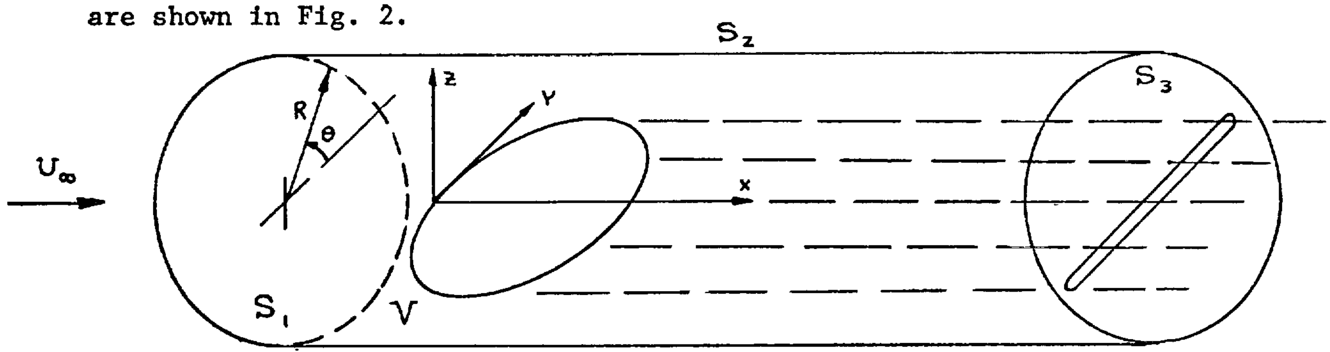

FIG. 2

The cylindrical surface 'S_2' is chosen far away from the body where disturbances have died out and the flow is parallel, and 'S_3' is far downstream of the wing. Recalling now that

$$\vec{q} = \phi_x\,\vec{i} + \phi_y\,\vec{j} + \phi_z\,\vec{k}\,,$$

The streamwise component of $\vec{F}_{body}$ is

$$D_i = -\iint_{S_3} (P - P_\infty)\, dS_3 - \rho_\infty\, U_\infty{}^2 \iint_{S_3} \phi_x(1 + \phi_x)\, dS_3\,. \tag{2.8}$$

The integrand of the first integral on the right hand side may be expressed in terms of the perturbantion velocity components as follows. Pressure coefficient is defined as

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho_\infty U_\infty{}^2} \, . \tag{2.9}$$

One form of Bernoulli's equation is

$$\frac{\partial \Phi}{\partial t} + \frac{1}{2}(Q^2 - U_\infty{}^2) + \int_{P_\infty}^{P} \frac{dP}{\rho} + (\Omega_\infty - \Omega) = 0 \, . \tag{2.10}$$

For the present case this results in

$$\frac{P - P_\infty}{\rho_\infty} = -\frac{1}{2}(Q^2 - U_\infty{}^2) \, . \tag{2.11}$$

Putting this into 2.9

$$C_p = 1 - \frac{Q^2}{U_\infty{}^2} \, , \tag{2.12}$$

or in terms of the perturbation velocity

$$\left.\begin{aligned} C_p &= 1 - (\vec{q} + \vec{i}) \cdot (\vec{q} + \vec{i}) \\ &= -(\phi_x{}^2 + \phi_y{}^2 + \phi_z{}^2 + 2\phi_x) \, . \end{aligned}\right\} \tag{2.13}$$

Combining this last result with 2.9 we get

$$P - P_\infty = -\frac{1}{2}\rho_\infty U_\infty{}^2(\phi_x{}^2 + \phi_y{}^2 + \phi_z{}^2 + 2\phi_x) \, . \tag{2.14}$$

Putting this into 2.8 we obtain

$$D_i = \frac{1}{2}\rho_\infty U_\infty{}^2 \iint_{S_3} (\phi_y{}^2 + \phi_z{}^2 - \phi_x{}^2) \, dS_3 \, . \tag{2.15}$$

Assuming a flat 2-D wake as is commonly done ($\phi_x = 0$)

$$D_i = \frac{1}{2} \rho_\infty \quad U_\infty^{\ 2} \iint_{S_3} (\phi_y^{\ 2} + \phi_z^{\ 2})\, dS_3 \, . \tag{2.16}$$

To facilitate evaluation of this integral we make use of the 2-D form of Green's theorem:

$$\iint_S [\phi_1 \nabla^2 \phi_2 + \nabla\phi_1 \cdot \nabla\phi_2]\, dS = - \oint_C \phi_1 \frac{\partial \phi_2}{\partial n}\, ds \, . \tag{2.17}$$

For $\phi_1 = \phi_2 = \phi$ and $\nabla^2\phi = 0$, this reduces to

$$\iint_S (\nabla\phi)^2\, dS = - \oint_C \phi \frac{\partial\phi}{\partial n}\, ds \, ,$$

or

$$\iint_S (\phi_y^{\ 2} + \phi_z^{\ 2})\, dS = - \oint_C \phi \frac{\partial\phi}{\partial n}\, ds \, . \tag{2.18}$$

Using this result in 2.16 we get

$$D_i = - \frac{1}{2} \rho_\infty \ U_\infty^{\ 2} \oint_C \phi \frac{\partial\phi}{\partial n}\, ds \, , \tag{2.19}$$

where "C" is the integration contour in the Trefftz plane as shown in Fig. 3.

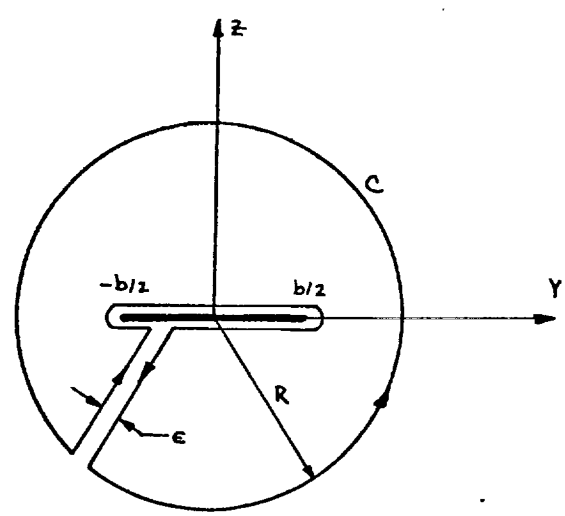

FIG. 3

In the limit of $R \to \infty$ and $\varepsilon \to 0$ the only contribution to the above integral comes from around the flat wake.

$$\left.\begin{aligned} D_i &= -\frac{1}{2}\rho_\infty U_\infty^{\;2} \oint_{-b/2}^{b/2} \phi_u \frac{\partial\phi}{\partial z}\,dy + \oint_{-b/2}^{b/2} \phi_\ell \frac{\partial\phi}{\partial z}\,dy \\ &= -\frac{1}{2}\rho_\infty U_\infty^{\;2} \oint_{-b/2}^{b/2} \Delta\phi \frac{\partial\phi}{\partial z}\,dy \\ &= -\frac{1}{2}\rho_\infty U_\infty^{\;2} \oint_{-b/2}^{b/2} (\Delta\phi)_{\substack{\text{wake}\\ x=\infty}} (w)_{\substack{x=\infty\\ z=0}}\,dy \end{aligned}\right\} \tag{2.20}$$

where the integrand consists of the product of ϕ-discontinuity across the wake and the vertical component of induced velocity at the wake, far downstream of the wing. Expressions for these are readily available in aerodynamic texts (e.g., see Ashley and Landahl, Section 7.3).

$$(\Delta\phi)_{\substack{\text{wake}\\ x=\infty}} = \Gamma(y)/U_\infty \tag{2.21}$$

$$(w)_{\substack{x=\infty\\ z=0}} = -\frac{1}{2\pi U_\infty} \oint_{-b/2}^{b/2} \frac{d\Gamma/dy_1}{y - y_1}\,dy_1 \,. \tag{2.22}$$

Putting these results into the last form of 2.20, we obtain the well known result

$$D_i = \frac{\rho_\infty}{4\pi} \oint_{-b/2}^{b/2} \oint_{-b/2}^{b/2} \Gamma(y) \quad \Gamma'(y_1) \frac{dy_1 dy}{y - y_1} \tag{2.23}$$

Making the transformation

$$y = (b/2)\cos\theta \tag{2.24}$$

and assuming a Glauert series for wing circulation distribution

$$\Gamma(\theta) = U_\infty \; b \sum_{n=1}^{\infty} A_n \sin(n\theta) \; , \tag{2.25}$$

the inner integral in 2.23 may now be evaluated using the Glauert integral.

$$\oint_{-b/2}^{b/2} \Gamma'(y_1) \frac{dy_1}{y-y_1} = 2\, U_\infty \sum_{n=1}^{\infty} n\, An \oint_0^{\pi} \frac{\cos(n\theta_1)}{\cos\theta_1 - \cos\theta}\, d\,\theta_1$$

$$= 2\,\pi\, U_\infty \sum_{n=1}^{\infty} n\, An \frac{\sin(n\theta)}{\sin\theta} \tag{2.26}$$

Putting 2.26 into 2.23 and making use of the properties of orthogonal functions we finally obtain

$$D_i = \frac{\pi}{8} \rho_\infty \; U_\infty^{\;2}\, b^2 \sum_{n=1}^{\infty} n\, An^2 \; . \tag{2.27}$$

Given the spanwise load distribution for a wing, one can determine 'A_n's from the theory of Fourier series, and Eq. 2.27 gives the induced or vortex drag for the wing. The form of 2.27 obviously suggests that the higher harmonics of the Glauert series add drag without increasing lift (which can be shown to be $L = \frac{\pi}{4} \rho_\infty \; U_\infty^{\;2}\, b^2\, A_1$). Hence minimum induced drag is achieved when only 'A_1' is present. This is the well known case of elliptic loading which is

$$\Gamma(y) = U_\infty \; b\, A_1 \sqrt{1 - y^2/(b/2)^2} \; . \tag{2.28}$$

The induced drag for this case is

$$D_{i_{min}} = \frac{\pi}{8} A_1^{\,2} \rho_\infty \; U_\infty^{\;2}\, b^2 \; . \tag{2.29}$$

2.2 Unsteady Flow—Streamwise Force on 2-D Oscillating Airfoil

The unsteady form of the momentum theorem presented in Chapter II gives the force experienced by a rigid body in unsteady motion.

$$\vec{F}_{body} = - \oint\!\!\oint_S [P \vec{n} + \rho\vec{Q}(\vec{Q} \cdot \vec{n})] \, dS - \iiint_V \frac{\partial}{\partial t} (\rho \vec{Q}) \, dV \tag{2.3}$$

Now, we desire to derive an expression for the streamwise force on the body in terms of the perturbation potential, for the case of a uniform, parallel, constant density flow (see Fig. 2). Introducing the perturbation velocity (see Eq. 2.5), the x-component of 2.3 becomes

$$F_x = - \iint_{S_3} (P - P_\infty) \, dS_3 - \rho U_\infty{}^2 \iint_{S_3} (\phi_x + \phi_x{}^2) \, dS_3$$

$$- \rho_\infty U_\infty \iiint_V \frac{\partial}{\partial t} \phi_x \, dV \, . \tag{2.30}$$

As in Section 2.1 we can use Bernoulli's equ. to express $(P - P_\infty)$ in terms of perturbation velocity components as shown below.

$$C_p = - \frac{2}{U_\infty} \phi_t + 1 - \frac{Q^2}{U_\infty{}^2}$$

$$= - [\phi_x{}^2 + \phi_y{}^2 + \phi_z{}^2 + 2 \phi_x + \frac{2}{U_\infty} \phi_t] \, .$$

Hence,

$$P - P_\infty = - \frac{1}{2} \rho_\infty U_\infty{}^2 [\phi_x{}^2 + \phi_y{}^2 + \phi_z{}^2 + 2 \phi_x + \frac{2}{U_\infty} \phi_t] \, . \tag{2.31}$$

Putting 2.31 into 2.30 we get

$$F_x = \frac{1}{2} \rho_\infty U_\infty{}^2 \iint_{S_3} [\phi_y{}^2 + \phi_z{}^2 - \phi_x{}^2 + \frac{2}{U_\infty} \phi_t] \, dS_3$$

$$- \rho_\infty U_\infty \iiint_V \frac{\partial}{\partial t} \phi_x \, dV \, . \tag{2.32}$$

Due to the complexity of the three dimensional problem it is advisable to work out the problem in two dimensions first. We are currently working on formulating the two dimensional problem to determine the streamwise force on a 2-D oscillating airfoil. The wake of this airfoil consists of a continuous band of spanwise vortices, of variable strength, shed from the trailing edge. No streamwise vortices, however, are present in the wake which renders the problem more manageable. The following is an outline of our current work on this problem.

1. Assume a thin airfoil at small angle of attack performing small amplitude oscillations in a uniform flow, producing a flat wake.
2. Since the flow field is governed by a linear equation ($\nabla^2\phi = 0$) we can use the principle of superposition to build up the potential from the potential of the individual vortices making up the wake.
3. Put the potential into 2.32 (with $\phi_y = 0$), average over time, and carry out the integration to obtain the average streamwise force on the airfoil.

The result obtained from the present method (momentum theorem) should be in agreement with the result obtained by von Karman and Burgers (see Ref. 3, p. 306). They directly calculate the force on a 2-D, plane, oscillating airfoil.

III. GENERAL METHOD

The following is a brief outline of the proposed method to determine an optimum full-span flapping wing configuration.

1. Equation 2.32 of the preceding section is an expression for the streamwise force experienced by an arbitrary aerodynamic configuration in three-dimensional unsteady flow.

$$F_x = \frac{1}{2}\rho_\infty U_\infty{}^2 \iint_{S_3} [\phi_y{}^2 + \phi_z{}^2 - \phi_x{}^2 + \frac{2}{U_\infty}\phi_t]\, dS_3$$

$$- \rho_\infty U_\infty \iiint_V \frac{\partial}{\partial t}\phi_x\, dV \qquad (3.1)$$

2. Assume a perturbation potential consisting of a steady part and an unsteady part of the following general form.

$$\phi(\bar{r}, t) = \phi_o(\bar{r}) + \phi(\bar{r})\, e^{i\omega(t - x/U_\infty)} \qquad (3.2)$$

3. Put 3.2 into 3.1 and average over time.
4. Use variational methods to determine a ' ϕ' that would optimize this average.
5. Knowing 'ϕ ' construct a wing configuration with proper time dependent displacements giving rise to such a potential.

It must be noted that the wake of the three dimensional flapping wing consists of a distribution of both spanwise and streamwise vortices, making the problem much more difficult than the two dimensional one. Appropriate simplifying assumptions will have to be formulated along the way to make the problem tractable.

IV. PROPOSED EXPERIMENTAL INVESTIGATIONS

There has been some discussion on the subject of experimental work dealing with the aerodynamics of bird flight. Thus far, this discussion has centered on the construction of a wind tunnel model of a three-dimensional flapping wing which would be tested in a low-speed air flow. The time variation of aerodynamic forces and moments would be measured and compared to theoretical predictions. The basic model configuration might be a wing of elliptic planform, flapping sinusoidally. The model would be of such size as to approximately correspond to the actual size of a medium to large bird's wing.

Perhaps the greatest foreseeable problem in the design and construction of such a model is the measurement of the desired forces and moments with sufficient accuracy to render the test results meaningful. Other aspects of design that deserve serious consideration are the relative merits of full-span and semi-span models and the exact nature of the flapping driving mechanisms.

Once an adequate flapping mechanism is built, the test possibilities are manifold. Models of different planform could be substituted for the nominal elliptic planform, including actual bird wing planforms. Velocity measurements in the wake and the application of flow visualization techniques could contribute substantially to the understanding of the geometry of the unsteady wake. (Present theoretical calculations make assumptions about the wake that render the mathematics manageable, but which may not, in fact, be justified.) A flexible or partially flexible model of a bird wing could be tested to explore the role aerodynamic compliance may play in determining the efficiency of bird flight. The possibilities are indeed open-ended requiring that each progressive phase of testing be carefully evaluated to facilitate the decision concerning the successive stages of investigation. Initial efforts will be directed toward building and testing a simple rigid flapping wing.

SYMBOLS

b	span
C_p	pressure coefficient
D_i	induced drag
$\vec{F}_{body}$	force on body
$\vec{i}$	unit vector in x-direction
$\vec{j}$	unit vector in y-direction
$\vec{k}$	unit vector in z-direction
L	lift
$\vec{n}$	unit normal vector
P	pressure
$\vec{q}$	perturbation velocity
$\vec{Q}$	total velocity
$\vec{r}$	position vector $(= x\vec{i} + y\vec{j} + z\vec{k})$
s	arc length
S	surface area
t	time
U	speed
V	volume
w	vertical component of velocity
x	streamwise coordinate
y	spanwise coordinate
z	vertical ccordinate

ρ	density
ϕ	perturbation velocity potential
Φ	total velocity potential
Γ	circulation
ω	frequency

Subscripts

$(\ )_\ell$	lower
$(\ )_u$	upper
$(\ )_\infty$	free stream conditions

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